

TITLE OF THE INVENTION  
HADAMARD TRANSFORMATION METHOD AND DEVICE

FIELD OF THE INVENTION

5           The present invention relates to an Hadamard transformation method and device capable of making the reversible transformation to output the integer data.

BACKGROUND OF THE INVENTION

10           An image, especially a multi-valued image contains a very large amount of information, and when the image is stored or transmitted, its data amount causes a problem. Therefore, to store or transmit the image, a high efficient coding is employed for coding the image data, in which the multi-valued image has the  
15           redundancy removed, or the content of the multi-valued image is changed to the extent that the image quality degradation is visually unrecognizable to reduce the data amount. For example, in a JPEG recommended by the  
20           ISO and ITU-T as the international standard coding system for still image, the image data is subjected to a Discrete Cosine Transformation (DCT) into DCT coefficients for each block (8 pixels  $\times$  8 pixels), each DCT coefficient being quantized, and further the  
25           quantized value is entropy coded to compress the image data. The compression techniques using this DCT include H261, MPEG1/2/4, in addition to the JPEG.

An Hadamard transformation is a process associated with this DCT transformation or a process for transforming the image data. The Hadamard transformation is an orthogonal transformation with a transformation matrix composed of the elements of "1" and "-1", which is the simplest one that is represented only by addition and subtraction.

For this Hadamard transformation, a transformation matrix  $H_2$  for two-point Hadamard transformation is defined in the following way.

[Expression 1]

$$H_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (1)$$

A typical  $N (=2^N)$ -point Hadamard transformation matrix  $H_N$  is recursively defined as a Kronecker product between the  $(N/2)$ -point Hadamard transformation matrix  $H_{N/2}$  and the two-point Hadamard transformation matrix  $H_2$ .

[Expression 2]

$$\begin{aligned} H_N &= H_{N/2} \otimes H_2 \\ &= \frac{1}{\sqrt{2}} \begin{bmatrix} H_{N/2} & H_{N/2} \\ H_{N/2} & -H_{N/2} \end{bmatrix} \end{aligned} \quad (2)$$

A four-point Hadamard transformation matrix is obtained from the above definition in the following way.

[Expression 3]

$$H_4 = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad (3)$$

This four-point Hadamard transformation matrix is called a natural type, in which the base vectors are not arranged in sequence. Thus, if the permutation of the base vectors is repeated to shift the base vector in the second row to the fourth row, a transformation matrix  $WH_4$  having the base vectors arranged in sequence results.

[Expression 4]

$$WH_4 = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \quad (4)$$

This transformation matrix  $WH_4$  is called a Walsh type or a Walsh-Hadamard transformation matrix. It is well known that the Hadamard transformation is a reversible orthogonal transformation, in which both the natural type and the Walsh type are capable of the reversible transformation, and the transformation matrix is symmetrical.

It is generally said that the Hadamard transformation is a reversible transformation as is described above, which means that it is mathematically reversible. That is, it is supposed that no

arithmetical operation error arises in the transformation and the reverse transformation. For this reversible transformation, it is required to perform the arithmetical operation in the fixed point  
5 and the floating point, and hold all the data of significant digits after the transformation processing.

However, in the Hadamard transformation in which the compression coding is supposed, there is a desire to reduce the number of significant digits as much as  
10 possible after the transformation processing. More specifically, the data after the decimal fractions that arises when the integer input data is transformed is grasped to have clearly more digits (information) than the input data, whereby it is indispensable to omit the  
15 data after the decimal fractions to compress the data. However, if this data after the decimal fractions is simply rounded, the reversible property is impaired. For example, four pieces of data, including 123, 78, 84 and 56, are subjected to the Hadamard transformation  
20 with a transformation matrix of the expression (4) in the following way.

$$170.5(=(123+78+84+56)/2), \quad 30.5(=(123+78-84-56)/2), \\ 8.5(=(123-78-84+56)/2), \quad 36.5(=(123-78+84-56)/2)$$

Rounding off the above values into the integer,  
25 171, 31, 9, 37

are obtained. Then, making the reverse transformation (with a transposed matrix of the transformation matrix, which is the same as the transformation matrix),

124, 78, 84, 56

5 are obtained. In this way, the initial data "123" is altered into "124" through the Hadamard transformation and its reverse transformation. Thus, the Hadamard transformation to output the integer data does not guarantee the reversibility. In the following, the  
10 Hadamard transformation to output the integer data is called an integer type Hadamard transformation, and the integer type Hadamard transformation capable of making the reversible transformation is called an integer type reversible Hadamard transformation or a lossless  
15 Hadamard transformation.

Conventionally, the lossless four-point Hadamard transformation was realized in the following way.

- (a) Decompose the four-point Hadamard transformation matrix  $H_4$  into a product of triangular matrices with  
20 the diagonal elements "1",
- (b) Apply a row replacing operation  $Q$  on the original transformation matrix, and
- (c) Decompose  $QH_4$  into a product of following matrices from the above (a) and (b).

25 [Expression 5]

$$QH_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & 1/2 & -1/2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \quad (5)$$

(d) Represent this transformation in a signal flow (as represented in FIG. 1),

(e) Round the data after the decimal fractions that arises in multiplication operation in this signal flow to make the integer data.

The lossless four-point Hadamard transformation is implemented with the above items (a) to (e). Some supplementary explanation is given in the following.

10        The signal flow of FIG. 1 has a Ladder Network configuration which is often employed to implement a reversible transformation process. In this Ladder Network, the transformation process for outputting the integer data is made reversible by introducing a

15        rounding process (200, 201 in FIG. 2) rounding the data to the output side of a multiplier 100, 101 in which the data after the decimal fractions is produced as a general method (in the field of reversible transformation process).

20        Herein, if the corresponding rounding process of the multiplication output is the same between the transformation process and the reverse transformation process, the content of the rounding process 200, 201 is arbitrary.

FIG. 2 shows a signal flow (capable of the reversible transformation) which is made reversible by introducing the rounding process into the signal flow of FIG. 1. This was a conventional arithmetical operation method for implementing the integer type reversible four-point Hadamard transformation.

A lossless 16-point Hadamard transformation is constructed by linking such lossless integer type reversible four-point Hadamard transformations at multistage. However, there is a dispersion in the average value of the error of each transformed output (a difference between mathematical transformation and lossless transformation) or the mean square error, giving rise to accurate transform coefficients and inaccurate transform coefficients .

Also, the conventional lossless two-dimensional DCT (Discrete Cosine Transformation) had a significant error of transformation with reference to the conventional two-dimensional DCT.

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#### SUMMARY OF THE INVENTION

The present invention has been achieved in the light of the above-mentioned conventional example, and the feature of the invention is to provide an Hadamard transformation method and device in which the content of rounding process is set up so that the mean error occurring in the lossless Hadamard transformation

process is theoretically zero to enhance the precision of transform coefficients .

According to the present invention, there is provided an Hadamard transformation method comprising:

5 a first transformation step of transforming input signals using a four-point Hadamard transformation matrix in each of four four-point Hadamard transformation units; a first rounding step of rounding up the least significant bit of each of the odd number

10 of four coefficients transformed by each of the four four-point Hadamard transformation units and discarding the least significant bit of each of the remaining odd number of the four coefficients, so as to produce the integer type of four sets of coefficients, each set

15 including four coefficients; a second transformation step of selecting one coefficient from each one set of the four sets such that odd numbers of coefficients among the four selected coefficients for one set were rounded up in the first rounding step, and supplying

20 the four selected coefficients to a four-point Hadamard transformation unit and transforming the four selected coefficients using a four-point Hadamard transformation matrix; and a second rounding step of rounding up the coefficients transformed in the second transformation

25 step so as to cancel propagation errors to be superposed over the transformed coefficients.



Other features and advantages of the present invention will be apparent from the following description taken in conjunction with the accompanying drawings, in which like reference characters designate  
5 the same or similar parts throughout the figures thereof.

#### BRIEF DESCRIPTION OF THE DRAWINGS

The accompanying drawings, which are incorporated  
10 in and constitute a part of the specification, illustrate embodiments of the invention and, together with the description, serve to explain the principles of the invention.

FIG. 1 depicts a diagram showing a signal flow  
15 that is a basis for implementing a conventional lossless  $4 \times 4$  Hadamard transformation process;

FIG. 2 depicts a diagram showing a signal flow of the conventional lossless  $4 \times 4$  Hadamard transformation process;

20 FIG. 3 depicts a block diagram showing the configuration of a lossless 16-point Hadamard transformation using lossless  $4 \times 4$  Hadamard transformations;

FIGS. 4A to 4H depict diagrams showing eight types  
25 of lossless four-point Hadamard transformation as already proposed by the present inventor;

FIG. 5 depicts a block diagram showing the configuration of a lossless 16-point Hadamard transformation according to a first embodiment of the invention;

5        FIGS. 6A to 6C are graphs showing a transformation error distribution in the lossless 16-point Hadamard transformation process according to the first embodiment of the invention;

FIG. 7 depicts a diagram showing one example of  
10    the configuration in which the mean error in transformation is not zero in contrast to the lossless 16-point Hadamard transformation of FIG. 5 according to the first embodiment of the invention;

FIGS. 8A to 8C are graphs showing the  
15    transformation error distribution in the configuration of FIG. 7;

FIG. 9 depicts a block diagram showing the configuration of a lossless two-dimensional Hadamard transformation according to a second embodiment of the  
20    invention;

FIG. 10 depicts a diagram showing the configuration of a lossless DCT as described in document 2;

FIG. 11 depicts a block diagram showing the  
25    configuration of a conventional lossless two-dimensional DCT;

FIG. 12 depicts a block diagram showing the configuration of a lossless two-dimensional DCT according to a third embodiment of the invention;

FIG. 13 depicts a diagram showing the  
5 configuration of a one-dimensional DCT that is effective for the lossless two-dimensional DCT according to the third embodiment of the invention; and

FIG. 14 depicts a table showing the comparison of the reverse transformation precision of the lossless  
10 two-dimensional DCT.

#### DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS

The preferred embodiments of the present invention will be described below with reference to the  
15 accompanying drawings.

[First Embodiment]

FIG. 3 is a block diagram showing the configuration of the lossless 16-point Hadamard transformation using lossless 4×4 Hadamard  
20 transformations.

As shown in FIG. 3, the eight lossless four-point Hadamard transform sections 300 to 307 are linked at multistage. Herein, 16 pieces of input data (d0 to d15) are divided into four blocks, and each block is  
25 subjected to the lossless four-point Hadamard transformation by each of the lossless four-point Hadamard transform sections 300 to 303. Then, four

transform coefficients at the same level in each transformation by the lossless four-point Hadamard transform sections 300 to 303 are gathered, and the four transform coefficients are subjected to the  
5 lossless four-point Hadamard transformation again by the lossless four-point Hadamard transform sections 304 to 307, respectively, to implement the lossless 16-point Hadamard transformation.

Herein, the lossless four-point Hadamard transform  
10 sections 300 to 307 may employ the Hadamard transformation configuration as shown in FIG. 2, or may employ lossless Hadamard transformations with any one of eight types of rounding process described later.

Explaining simply, the eight types of the rounding  
15 process comprise rounding up the odd number of four transform coefficients and rounding down the remaining odd number of the four transform coefficients. In other words, one or three the four transform coefficients are rounded up and the remaining  
20 coefficients are rounded down.

FIGS. 4A to 4H depict diagrams showing the processings in eight types of lossless four-point Hadamard transform sections. Herein, the transformation process with the Hadamard transformation  
25 matrix according to the expression (3) or (4) is performed by each of the four-point Hadamard transform sections 410 to 417 at the former stage, and the

rounding process is performed by 400 to 407 at the latter stage. Herein, the content of the rounding process is simply indicated as "+" and "-", in which "+" means the round-up and "-" means the round-down (discard). The round-up and round-down as used in this specification means the rounding up and discarding process on two's complement representation, but not the absolute value representation. For example, a complement representation of "-7.5" is "11111000.1", in which if this fraction part is rounded up, "11111001" (-7) results, and if it is rounded down, "11111000" (-8) results.

The reverse transformation is also one kind of lossless transformation. Its reason is that if the data after reverse transformation is transformed again, the data becomes equal to the data before the reverse transformation. Hence, the eight types of lossless four-point Hadamard transform sections 410 to 417, 400 to 407 as shown in FIGS. 4A to 4H shows also all types of reverse transformation. The correspondence between transformation and reverse transformation is different depending on the type of Hadamard transformation matrix (natural type or Walsh type).

The natural type has the following relationship between the transformation and the reverse transformation as indicated by drawing numbers.

$$(4A) \rightarrow (4A);$$

(4B)  $\rightarrow$  (4G);  
 (4C)  $\rightarrow$  (4F);  
 (4D)  $\rightarrow$  (4H);  
 (4E)  $\rightarrow$  (4E);  
 5 (4F)  $\rightarrow$  (4C);  
 (4G)  $\rightarrow$  (4B);  
 (4H)  $\rightarrow$  (4D)

Also, the Walsh type has the following  
 relationship between the transformation and the reverse  
 10 transformation as indicated by drawing numbers.

(4A)  $\rightarrow$  (4A);  
 (4B)  $\rightarrow$  (4F);  
 (4C)  $\rightarrow$  (4H);  
 (4D)  $\rightarrow$  (4G);  
 15 (4E)  $\rightarrow$  (4E);  
 (4F)  $\rightarrow$  (4B);  
 (4G)  $\rightarrow$  (4D);  
 (4H)  $\rightarrow$  (4C)

In natural type or Walsh type, the rounding  
 20 process is adapted to cancel a propagation error in the  
 reversed transformed data.

In the following description, the transformation  
 matrix of Walsh type is employed, unless specifically  
 noted.

25 The lossless 4x4 Hadamard transformation using any  
 of the eight types of the rounding process as shown in  
 FIGS. 4A to 4H may be equally applied in FIG. 3 to make

the lossless 16-point Hadamard transformation. There are  $8^8$  kinds of combinations. Thus, the following issues are imposed.

(A1) Make the mean error for each transform coefficient  
5 zero.

The random number without correlation is assumed as the input data. The lossless 16-point Hadamard transformation satisfying the issue (A1) is configured on the basis of the following conditions (B1) and (B2).

10 (B1) For the four intermediate data input into each of the Hadamard transform sections 304 to 307 at the second stage (latter stage), the rounding process at the first stage is decided such that the odd number of data are rounded up, and the remaining odd number of  
15 data are rounded down. Then, the rounding process at the first stage is of course one of the eight types of rounding process.

(B2) Each of the Hadamard transform sections 304 to 307 at the second stage decides the type of rounding  
20 process to make the reverse transformation, grasping four input data as being generated from the same Hadamard transformation.

Of the four Hadamard transform sections 300 to 303 at the first stage, up to three Hadamard transform  
25 sections can freely decide the rounding process, but the remaining one has the rounding process uniquely decided if the condition (B1) is satisfied. Further,

the four Hadamard transform sections 304 to 307 at the second stage uniquely decided if the condition (B2) is satisfied. In this way, the configuration to satisfy the conditions (B1) and (B2) is simply obtained, and  
5 has the 512 ( $= 8^3$ ) types.

FIG. 5 depicts a block diagram showing the configuration of the lossless 16-point Hadamard transformation according to the first embodiment of the invention. The four-point Hadamard transform sections  
10 501 to 504 and 521 to 524 in FIG. 5 have the same configuration as the four-point Hadamard transform sections 410 to 417 as shown in FIGS. 4A to 4H.

In FIG. 5, "+" in the rounding processes 511 to 514 and 531 to 534 denotes the round up, and "-"  
15 " denotes the discard. In this example, seven types of rounding operation are employed among all the eight types of operations. When configured in this way, the mean error of arbitrary output becomes not only "0", but also the error distribution is the same, whereby  
20 the mean square error becomes equal. In the following, the reason why the mean error becomes "0" through the error distribution will be described.

In FIG. 5, four inputs (intermediate data) for each of the Hadamard transform sections 521 to 524 at  
25 the second stage have no correlation because they are not supplied from the same Hadamard transform section. For example, a superposed error in the input data into



the Hadamard transform section 521 occurs at the same probability in all sixteen cases from (0, 0, 0, 0) to (+0.5, -0.5, -0.5, -0.5). In FIG. 5, the intermediate data 540 is the rounded up data of a transformed  
5 coefficient and contains a round-up error (+0.5) at a probability of 1/2, and the transformed coefficient is the integer and error free data at a probability of 1/2. For other three intermediate data 541 to 543, the rounding process is the discard, in which an error (-  
10 0.5) due to the discard occurs at a probability of 1/2 in each intermediate data, and each transformed coefficient is the integer and error free data at a probability of 1/2 in the same way as above.

The values which are the product of 1/2 and the  
15 inner product between this superposed error and the first row of the transformation matrix in the expression (4), are distributed from the minimum "-0.75" to the maximum "0.25" at a step of "0.25". More particularly, the values of "-0.75", "-0.5", "-0.25",  
20 "0", and "+0.25" are distributed at the probabilities of "1/16", "1/4", "3/8", "1/4" and "1/16", respectively. FIG. 6A shows this distribution. If this distribution has "+0.5" added in a rounding process 531 after transformation process by the Hadamard transform  
25 section 521 at the second stage, the error distribution is as shown in FIG. 6B.

In this case, the probability that "+0.5" is added in the rounding process 531 after transformation process by the Hadamard transform section 521 at the second stage in FIG. 5 becomes equal to the probability  
5 that the least significant bit becomes "1", which is equal to 1/2.

Accordingly, the distributions as shown in FIGS. 6A and 6B arise at the same probability of "1/2", respectively, and the overall error distribution is as  
10 shown in FIG. 6C. Also, the mean square error of this distribution is "0.125".

The values which are the product 1/2 and the inner product between the superposed error and the second, third and fourth row of the transformation matrix in  
15 the expression (4), are distributed from "-0.25" to "0.75" at a step of "0.25", in contrast to the distribution of FIG. 6A. Since the value added in the rounding process after the Hadamard transform section at the second stage is symmetrically "-0.5", the  
20 overall error distribution is symmetrical, resulting in the same distribution. The input data for other three Hadamard transformations have the same error distribution using the same calculation, though the distribution of superposed error is slightly different,  
25 whereby the mean error is "0" and the mean square error is "0.125". Consequently, the mean error becomes "0"

and the mean square error becomes "0.125" in all the outputs y0 to y15 of FIG. 5.

For reference, the configuration not satisfying the above condition (B1) is shown in FIG. 7, and the behavior of the mean error and the mean square error will be described below. The wirings in FIG. 7 are the same as in FIG. 5, but the rounding process is different.

In FIG. 7, since four input data of the Hadamard transform section 521 at the second stage are all the rounded up data, its superposed error is deviated in + direction such as (0, 0, 0, 0) to (+0.5, +0.5, +0.5, +0.5). Therefore, the values which are the product of 1/2 and the inner product between the superposed error and the first row of the transformation matrix in the expression (4), have a distribution as shown in FIG. 8A. Thereafter, if "+0.5" is further added in a rounding process 711, the error distribution is as shown in FIG. 8B, and the overall error distribution is as shown in FIG. 8C. Hence, the mean error of output y0 becomes "0.75", and the mean square error becomes "0.6875". However, the error distribution is the same as in FIG. 6C, with its variance being "0.125". Herein, the mean square error, which is  $\{(\text{mean error})^2 + \text{variance}\}$ , becomes larger because the mean error is not "0".

In FIG. 7, the error superposed on any of the outputs y0 to y15 has the same distribution, with its

variance being "0.125". In this way, the mean error is different depending on the output, and the mean square error is varied accordingly. All the outputs are classified into the following three groups, according  
5 to the mean error and the mean square error.

Mean error = 0.75, mean square error = 0.6875 : y0

Mean error = 0.25, mean square error = 0.1875 : y7,  
y8, y15

Mean error = -0.25, mean square error = 0.1875 :  
10 others

If the lossless 16-point Hadamard transformation is configured without satisfying the above condition (B1), the errors are superposed as above example.

The lossless 16-point Hadamard transformation  
15 according to this embodiment can be easily implemented by the software processing. Each Hadamard transform section of real type can be implemented with the fixed point operation or floating point operation, and the rounding process can be performed in such a way that  
20 for rounding up, "0.5" is added and the floor function is acquired, and for rounding down, the floor function is acquired without addition, whereby the equivalent operation for the configuration of FIG. 5 can be made by the software processing.

25 [Second Embodiment]

In the lossless 16-point Hadamard transformation implemented in the first embodiment, in a case where

the input data is regularly arranged in a 4x4 matrix,  
the lossless two-dimensional Hadamard transformation is  
realized for the 4x4 two-dimensional data. As a  
preparation before showing the configuration for the  
5 two-dimensional process, the mathematical notation is  
simplified as shown in the following expression (6).

[Expression 6]

$$\begin{bmatrix} + & - & - & - \\ - & + & - & - \\ - & - & + & - \\ - & - & - & + \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} d0 & d4 & d8 & d12 \\ d1 & d5 & d9 & d13 \\ d2 & d6 & d10 & d14 \\ d3 & d7 & d11 & d15 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \times \frac{1}{2} \begin{bmatrix} + & - & - & - \\ + & - & + & + \\ + & + & + & - \\ + & + & - & + \end{bmatrix} \quad (6)$$

In this expression (6), the first (leftmost) and  
10 the fifth (rightmost) matrices composed of "+" and "-  
" signs represent the contents of the rounding process.  
Herein, each symbol has the same meaning as "+" and "-  
" in FIGS. 4A to 4H. First, the arithmetical operation  
is made between the second and third matrices from the  
15 left, the rounding process as defined in the first  
matrix is performed after multiplying the factor 1/2.  
Then, the arithmetical operation with the fourth matrix  
is made, and the rounding process of the last matrix is  
made after multiplying the factor 1/2. In the non-  
20 linear arithmetical operation, if the order of  
operation is changed, the characteristics are changed.  
In this case, unless the rounding operation for the  
leftmost matrix is firstly performed, the mean error  
becomes non "0".

The transformation in the expression (6) is one example, and there are 512 types of transformations in which the mean error becomes "0" as already described. There is a difference only between the leftmost and  
5 rightmost matrices representing the content of the rounding process. Hence, the lossless 4x4 two-dimensional Hadamard transformation (lossless 16-point Hadamard transformation) are defined by expressing the two matrices.

10 On the other hand, the configuration for implementing this lossless 16-point Hadamard transformation two-dimensionally is shown in FIG. 9. Herein, every four data is processed in a vertical direction, and its results are transposed and processed  
15 in a horizontal direction. The order of processing may be reversed.

FIG. 9 depicts a block diagram showing the configuration of the lossless two-dimensional Hadamard transformation according to the second embodiment of  
20 the invention.

In FIG. 9, numerals 901 and 911 denote Hadamard transform sections for making the Hadamard transformation process on the basis of the transformation matrix of the expression (3) or (4).  
25 Numeral 903 and 913 denote decoders for generating the addition data to be rounded up in accordance with the column number and the row number of the two-dimensional

data to be processed. Numerals 905 and 915 denote  
adder sections for making the adding process for  
rounding up. Numeral 907 and 917 denote discard  
sections for discarding the fraction part, and numeral  
5 909 denotes a buffer for temporarily storing the 4x4  
two-dimensional data to be transposed (rearranged).

The discard sections 907 and 917 for discarding  
the fraction part are provided on the process concept,  
but actually implemented by disconnecting the signal  
10 line of the fraction part and outputting the integer  
part only.

First of all, four data in the first column are  
input in parallel into the Hadamard transform section  
901. Herein, the transformation process is performed  
15 in accordance with the transformation matrix of the  
expression (3) or (4). Since the rounding process on  
the reverse transformation side is varied by the  
transformation matrix, the Walsh type Hadamard  
transformation matrix of the expression (4) is employed.  
20 The data transformed in this Hadamard transform section  
901 is the data having the fraction part of one bit,  
and passed to the adder section 905.

On the other hand, the decoder 903 generates the  
addition data for rounding up on the basis of the input  
25 column number information, and supplies it to the adder  
section 905. That is, the addition data (1, 0, 0, 0),  
(0, 1, 0, 0), (0, 0, 1, 0) and (0, 0, 0, 1) are

generated for "1", "2", "3" and "4" that are input as the column number and passed to the adder section 905. These addition data correspond to the column vector of the matrix at the leftmost location in the expression  
5 (6), in which "+" and "-" are replaced with "1" and "0", respectively.

The addition data for rounding up is added to the fraction part of the transformed data in the adder section 905. Hence, the weight of the addition data is  
10 set to "0.5". And the next discard section 907 discards the fraction part, whereby the rounding process is completed. Herein, when the fraction part of the transformed data is "0", the rounding process performed in the adder section 905 and the discard  
15 section 907 has no influence on the transformed data. However, when the fraction part of the transformed data is "1", the transformed data to which "0.5" is added is rounded up, and the other data is discarded.

If the column numbers of four data to be processed  
20 are changed, the addition data for rounding up is correspondingly changed, whereby the rounding process for each column number is changed. In this way, the data that has been subjected to the lossless Hadamard transformation is saved in the buffer 909. This buffer  
25 909 is so small as to transpose the 4x4 data, and may be easily constructed by a register, for example.



If the lossless transformation process for four columns (column number 4) of data is ended, the data is transposed in the buffer 909 and output as the row data. This row data is subjected to the lossless Hadamard transformation, employing the four-point Hadamard transform section 911, the decoder 913, the adder 915 and the discard section 917. The decoder 913 outputs the addition data corresponding to the row vector in the rightmost matrix of the expression (6). Thereby, the mean error of transformed data after the lossless two-dimensional Hadamard transformation becomes "0", and the mean square error becomes "0.125".

[Modification to the Second Embodiment]

The two adder sections 905 and 915 may be configured to output different data depending on the value of the least significant bit (equal for four data) of the four data output from the corresponding Hadamard transform sections 901 and 911. That is, the adder sections 905 and 915 receives the least significant bit, and outputs the addition data of four "0" when the least significant bit is "0", or outputs "0.5" or "-0.5" when the least significant bit is "1". In other words, the adder sections 904 and 915 receives the least significant bit (L), and outputs  $(0.5 \times L)$  or  $(-0.5 \times L)$ . The same process as the rounding process as previously described may be performed for the output from the adder sections 905 and 915.

[Third embodiment]

This third embodiment is the lossless two-dimensional DCT (Discrete Cosine Transformation), employing the 4x4 lossless two-dimensional Hadamard transformation as described in the second embodiment. The lossless DCT was firstly realized in the following document 1. (Document 1) Kuninori Komatsu and Kaoru Sezaki, "Reversible Discrete Cosine Transformation and its Application to Image Information Compression", Shingaku Giho IE97-83, p.1-6, Nov. 1997.

In this document 1, it is pointed out that because the Hadamard transformation is not used, a great number of multiplications are performed. As seen from the transformation matrix of the expression (3) or (4), the four-point Hadamard transformation is implemented only by addition and subtraction and the bit shift, and has no need for the multiplication. Though the rounding process is increased with the lossless transformation, it is only necessary to perform the addition to deal with the rounding up process, using the arithmetical operation method according to this invention.

Also, the lossless DCT using the lossless four-point Hadamard transformation has been offered in the following document 2. The lossless four-point Hadamard transformation as used herein has the configuration of FIG. 2. (Document 2) Shinji Fukuma, Koichi Oyama, Masahiro Iwabashi and Noriaki Kamibayashi, "Lossless 8-

point Fast Discrete Cosine Transformation Applying  
Lossless Hadamard Transformation", Shingaku Giho IE99-  
65, p.1-6, Dec. 1999.

FIG. 10 depicts a block diagram showing the  
5 configuration of the lossless DCT as described in this  
document 2.

In FIG. 10, reference numerals 1001, 1003 and 1005  
denote the lossless four-point Hadamard transform  
sections, and the two-input and two-output blocks 1006  
10 to 1010 are rotation units for rotating the two-  
dimensional vector on the plane. The rotation units  
1006 to 1010 are configured as lossless rotation units  
with a Ladder Network as described in the document 2.  
All the units are made lossless (reversible), whereby  
15 the entire DCT becomes lossless (reversible) to be able  
to make the lossless DCT process. This configuration  
is only shown with the one-dimensional transformation.

When the two-dimensional DCT is performed, it is  
common that the one-dimensional DCT is processed  
20 separately in the horizontal and vertical directions,  
as shown in FIG. 11.

FIG. 11 depicts a block diagram showing the  
configuration of a conventional lossless two-  
dimensional DCT.

25 In this arithmetical operation order, the lossless  
two-dimensional Hadamard transformation is not  
performed. Its reason is that a horizontal Hadamard

transformation process 1100 and a vertical Hadamard transformation process 1101 are separated. Therefore, it is almost meaningless that the mean error is made zero. In the arithmetical operation order of the two-dimensional DCT in FIG. 11, the latter process in the horizontal direction and the lossless Hadamard transformation process in the vertical direction may be exchanged, and performed in the order as shown in FIG. 12, enabling the complete two-dimensional Hadamard transformation to be made, whereby the new lossless two-dimensional DCT using the lossless two-dimensional Hadamard transformation is implemented.

FIG. 12 depicts a diagram showing the configuration of a lossless two-dimensional DCT according to a third embodiment of the invention.

FIGS. 11 and 12 show the software process in the processing order, in which these figures are not in the flowchart format but closest to the flowchart, and the corresponding flowcharts are omitted.

Though the configuration as shown in FIG. 10 enables the process to be made in the order as shown in FIG. 12, it is also possible to make the process with the configuration as shown in FIG. 13 in the order of FIG. 12.

FIG. 13 depicts a diagram showing the configuration of the one dimensional DCT that is effective for the lossless two-dimensional DCT of FIG.

12. In FIG. 13, the lossless Hadamard transformation process with two lossless four-point Hadamard transform sections at the former stage employs the technique as described in the document 2, while the lossless  
5 transformation process at the latter stage includes a slight variation of the technique of the document 1. When the process is performed in the order of FIG. 12, a configuration of FIG. 13 has a great effect. In evaluating the transformation precision as will be  
10 described later, the configuration of FIG. 13 is employed.

The transformation precision of the lossless DCT is problematical in respect of the compatibility. That is, the code compressed using the lossless DCT is  
15 decoded by the decoder using the conventional lossy DCT, there occurs a signal degradation due to miss match of DCT besides the compression. And this signal degradation is more predominant for the higher image quality compression with higher compression ratio.

20 FIG. 14 depicts a table showing the reverse transformation precision of the lossless two-dimensional DCT according to this embodiment. For reference, the precisions of the lossless two-dimensional DCTs as described in the documents 1 and 2  
25 are shown for comparison. The precision as shown in FIG. 14 is calculated on the basis of the specification

of reverse transformation precision for the DCT in the ITU-T H. 261 standard.

As will be clear from FIG. 14, in the lossless two-dimensional DCT according to this embodiment, the total pixel mean value of mean square errors and the maximum value for the pixels are closer as compared with the methods of the above documents. This means that there is a less dispersion in the mean square error between pixels, and all the pixels are degraded almost uniformly. The total pixel mean value of the mean square errors in this embodiment is decreased to about "2/3" to "1/2" as compared with the documents 1 and 2, but the maximum value of the mean square error is decreased to about "1/3" to "1/3.7" as compared with the documents 1 and 2. Moreover, the mean error has the similar trend. That is, in this embodiment, the total pixel mean value is doubled or more as compared with the document 1, but the maximum value is conversely decreased to about 60%.

In this way, with this embodiment, the precision is improved for all the items, except for the mean error over all the pixels.

In the embodiments, examples of the two-dimensional DCT using the two-dimensional Hadamard transformation are described, but upon two-dimensional Discrete Cosine Transforming 8x8 block data, the 8x8 block data may be segmented into four 4x4 block data

and each 4x4 block data may be two-dimensional transformed using the two-dimensional Hadamard transformation.

This invention may be applied to the system  
5 consisting of a plurality of apparatuses (e.g., host computer, interface unit, reader, and printer), or to a single apparatus (e.g., copying machine, and facsimile).

In the embodiments, the Hadamard transformations are implemented using a hardware, but those may be  
10 implemented using a software.

Also, the object of the invention is achieved by supplying a storage medium (recording medium) recording a software program code for implementing the functions of the embodiment to the system or apparatus, and  
15 reading and executing the program code stored in the storage medium in a computer (or CPU or MPU) of the system or apparatus. In this case, the program code itself read from the storage medium implements the functions of the embodiment, and the storage medium  
20 storing the program code constitutes the invention. The functions of the embodiment are implemented when the computer reads and executes the program code. However, the functions of the embodiment may be also implemented when an operating system (OS) of the  
25 computer performs a part or all of the process upon an instruction of the program code.

Moreover, the functions of the embodiment may be implemented in such a way that the program code read from the storage medium is written into a function extension card inserted into the computer or a memory  
5 equipped for the function extension unit connected to the computer, and the CPU provided for the function extension card or function extension unit performs a part or all of the actual process upon an instruction of the program code.

10 As described above, according to the first embodiment, to construct the lossless 16-point Hadamard transformation, using the 4x2 stage lossless four-point Hadamard transformation, the rounding process at the first stage is configured such that for four  
15 intermediate data inputted into the Hadamard transformation at the second stage, the odd number of data is rounded up, and the remaining odd number of data is rounded down, and the rounding process at the second stage is configured such that the Hadamard  
20 transformation at the second stage makes the reverse transformation, grasping four input data as coming from the same Hadamard transformation.

Thereby, the mean error of transform coefficients after the lossless transformation can be "0". Also,  
25 the lossless two-dimensional Hadamard transformation for 4x4 data in which the mean error of transform coefficients becomes "0" is realized.



Moreover, if the 4x4 lossless two-dimensional Hadamard transformation is performed in the arithmetical operation process of the lossless two-dimensional DCT, the transformation precision of the lossless two-dimensional DCT can be improved over the conventional method.

Also, in the two-dimensional lossless transformation device, the rounding process may be changed depending on the row or column number of the two-dimensional data to be processed, whereby the transformation precision is improved.

Moreover, if the lossless two-dimensional Hadamard transformation is performed in the arithmetical operation process of the lossless two-dimensional DCT, the lossless two-dimensional DCT with high transformation precision can be realized.

The present invention is not limited to the above embodiments and various changes and modifications can be made within the spirit and scope of the present invention. Therefore to apprise the public of the scope of the present invention, the following claims are made.